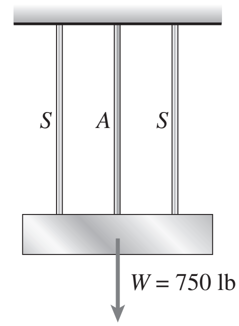
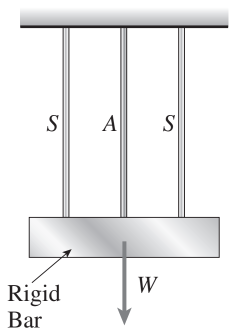
Problem 2.5-3 A rigid bar of weight $W = 750$ lb hangs from three equally spaced wires, two of steel and one of aluminum (see figure). The diameter of the wires is ⅛ in. Before they were loaded, all three wires had the same length.

What temperature increase ΔT in all three wires will result in the entire load being carried by the steel wires? (Assume $E_s = 30 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}$/°F, and $\alpha_a = 12 \times 10^{-6}$/°F.)



Solution 2.5-3 Bar supported by three wires



S = steel A = aluminum

$W = 750$ lb

$$d = \frac{1}{8} \text{ in.}$$

$$A_s = \frac{\pi d^2}{4} = 0.012272 \text{ in.}^2$$

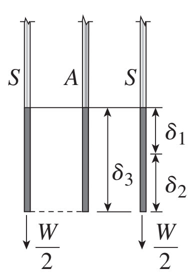
$E_s = 30 \times 10^6$ psi

$E_s A_s = 368{,}155$ lb

$\alpha_s = 6.5 \times 10^{-6}$/°F

$\alpha_a = 12 \times 10^{-6}$/°F

L = Initial length of wires



δ_1 = increase in length of a steel wire due to temperature increase ΔT

$= \alpha_s (\Delta T)L$

δ_2 = increase in length of a steel wire due to load $W/2$

$$= \frac{WL}{2E_s A_s}$$

δ_3 = increase in length of aluminum wire due to temperature increase ΔT

$= \alpha_a(\Delta T)L$

For no load in the aluminum wire:

$\delta_1 + \delta_2 = \delta_3$

$$\alpha_s(\Delta T)L + \frac{WL}{2E_s A_s} = \alpha_a(\Delta T)L$$

or

$$\Delta T = \frac{W}{2E_s A_s(\alpha_a - \alpha_s)} \longleftarrow$$

Substitute numerical values:

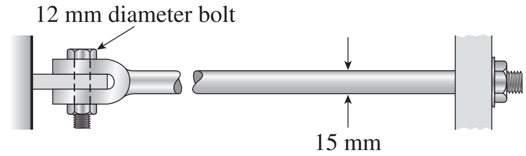
$$\Delta T = \frac{750 \text{ lb}}{(2)(368{,}155 \text{ lb})(5.5 \times 10^{-6}/°\text{F})}$$

$= 185°$F ⟵

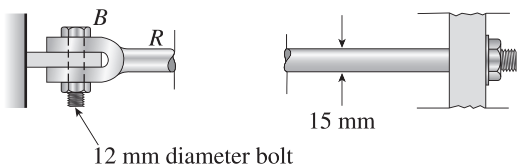
NOTE: If the temperature increase is larger than ΔT, the aluminum wire would be in compression, which is not possible. Therefore, the steel wires continue to carry all of the load. If the temperature increase is less than ΔT, the aluminum wire will be in tension and carry part of the load.

Problem 2.5-4 A steel rod of diameter 15 mm is held snugly (but without any initial stresses) between rigid walls by the arrangement shown in the figure.

Calculate the temperature drop ΔT (degrees Celsius) at which the average shear stress in the 12-mm diameter bolt becomes 45 MPa. (For the steel rod, use $\alpha = 12 \times 10^{-6}/°C$ and $E = 200$ GPa.)



Solution 2.5-4 **Steel rod with bolted connection**



R = rod

B = bolt

P = tensile force in steel rod due to temperature drop ΔT

A_R = cross-sectional area of steel rod

From Eq. (2-17) of Example 2-7: $P = EA_R\alpha(\Delta T)$

Bolt is in double shear.

V = shear force acting over one cross section of the bolt

$$V = P/2 = \frac{1}{2}EA_R\alpha(\Delta T)$$

τ = average shear stress on cross section of the bolt

A_B = cross-sectional area of bolt

$$\tau = \frac{V}{A_B} = \frac{EA_R\alpha(\Delta T)}{2A_B}$$

Solve for ΔT: $\Delta T = \dfrac{2\tau A_B}{EA_R\alpha}$

$$A_B = \frac{\pi d_B^2}{4} \quad \text{where } d_B = \text{diameter of bolt}$$

$$A_R = \frac{\pi d_R^2}{4} \quad \text{where } d_R = \text{diameter of steel rod}$$

$$\Delta T = \frac{2\tau d_B^2}{E\alpha d_R^2} \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\tau = 45$ MPa $\quad d_B = 12$ mm $\quad d_R = 15$ mm

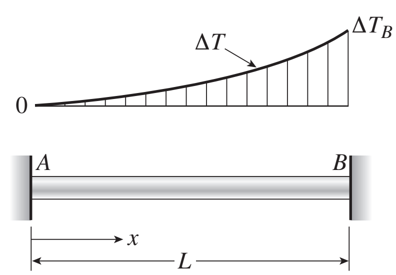
$\alpha = 12 \times 10^{-6}/°C \quad E = 200$ GPa

$$\Delta T = \frac{2(45\text{ MPa})(12\text{ mm})^2}{(200\text{ GPa})(12 \times 10^{-6}/°C)(15\text{ mm})^2}$$

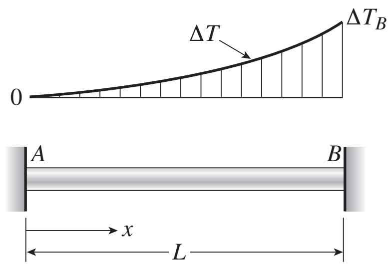
$$\Delta T = 24°C \longleftarrow$$

Problem 2.5-5 A bar AB of length L is held between rigid supports and heated nonuniformly in such a manner that the temperature increase ΔT at distance x from end A is given by the expression $\Delta T = \Delta T_B x^3/L^3$, where ΔT_B is the increase in temperature at end B of the bar (see figure).

Derive a formula for the compressive stress σ_c in the bar. (Assume that the material has modulus of elasticity E and coefficient of thermal expansion α.)



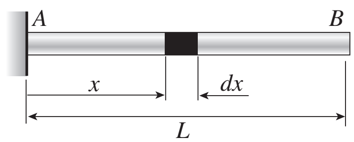
Solution 2.5-5 Bar with nonuniform temperature change



At distance x:

$$\Delta T = \Delta T_B \left(\frac{x^3}{L^3}\right)$$

Remove the support at end B of the bar:



Consider an element dx at a distance x from end A.

$d\delta$ = Elongation of element dx

$$d\delta = \alpha(\Delta T)dx = \alpha(\Delta T_B)\left(\frac{x^3}{L^3}\right)dx$$

δ = elongation of bar

$$\delta = \int_0^L d\delta = \int_0^L \alpha(\Delta T_B)\left(\frac{x^3}{L^3}\right)dx = \frac{1}{4}\alpha(\Delta T_B)L$$

Compressive force P required to shorten the bar by the amount δ

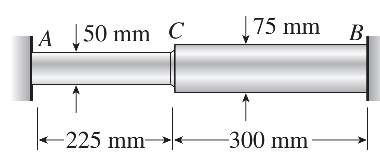
$$P = \frac{EA\delta}{L} = \frac{1}{4}EA\alpha(\Delta T_B)$$

Compressive stress in the bar

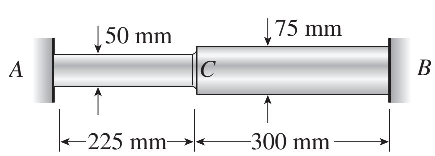
$$\sigma_c = \frac{P}{A} = \frac{E\alpha(\Delta T_B)}{4} \quad \longleftarrow$$

Problem 2.5-6 A plastic bar ACB having two different solid circular cross sections is held between rigid supports as shown in the figure. The diameters in the left- and right-hand parts are 50 mm and 75 mm, respectively. The corresponding lengths are 225 mm and 300 mm. Also, the modulus of elasticity E is 6.0 GPa, and the coefficient of thermal expansion α is 100×10^{-6}/°C. The bar is subjected to a uniform temperature increase of 30°C.

Calculate the following quantities: (a) the compressive force P in the bar; (b) the maximum compressive stress σ_c; and (c) the displacement δ_C of point C.



Solution 2.5-6 Bar with rigid supports



$E = 6.0$ GPa $\qquad \alpha = 100 \times 10^{-6}$/°C

Left-hand part:

$L_1 = 225$ mm $\qquad d_1 = 50$ mm

$$A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(50 \text{ mm})^2$$

$$= 1963.5 \text{ mm}^2$$

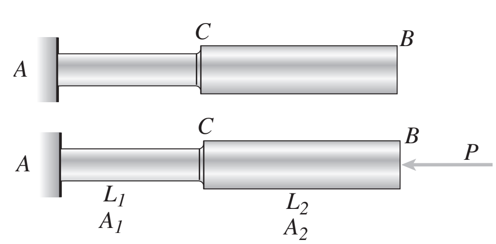
$\Delta T = 30$°C

Right-hand part:

$L_2 = 300$ mm $\qquad d_2 = 75$ mm

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(75 \text{ mm})^2 = 4417.9 \text{ mm}^2$$

(a) Compressive force P



Remove the support at end B.

δ_T = elongation due to temperature

$P = \alpha(\Delta T)(L_1 + L_2)$

$= 1.5750$ mm

δ_P = shortening due to P

$$= \frac{PL_1}{EA_1} + \frac{PL_2}{EA_2}$$

$= P(19.0986 \times 10^{-9}\text{ m/N} + 11.3177 \times 10^{-9}\text{ m/N})$

$= (30.4163 \times 10^{-9}\text{ m/N})P$

(P = newtons)

Compatibility: $\delta_T = \delta_P$

$1.5750 \times 10^{-3}\text{ m} = (30.4163 \times 10^{-9}\text{ m/N})P$

$P = 51{,}781$ N or P = 51.8 kN ⟵

(b) Maximum compressive stress

$$\sigma_c = \frac{P}{A_1} = \frac{51.78\text{ kN}}{1963.5\text{ mm}^2} = 26.4\text{ MPa} \quad \longleftarrow$$

(c) Displacement of point C

δ_C = Shortening of AC

$$\delta_C = \frac{PL_1}{EA_1} - \alpha(\Delta T)L_1$$

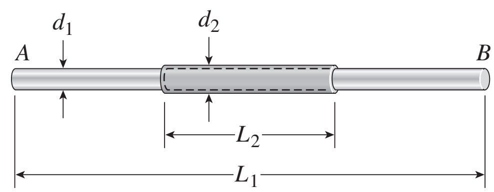
$= 0.9890\text{ mm} - 0.6750\text{ mm}$

$\delta_C = 0.314$ mm ⟵

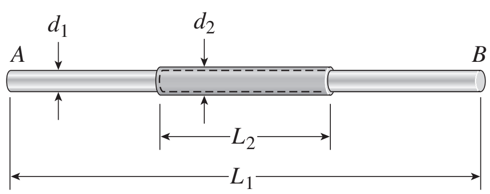
(Positive means AC shortens and point C displaces to the left.)

Problem 2.5-7 A circular steel rod AB (diameter $d_1 = 1.0$ in., length $L_1 = 3.0$ ft) has a bronze sleeve (outer diameter $d_2 = 1.25$ in., length $L_2 = 1.0$ ft) shrunk onto it so that the two parts are securely bonded (see figure).

Calculate the total elongation δ of the steel bar due to a temperature rise $\Delta T = 500°$F. (Material properties are as follows: for steel, $E_s = 30 \times 10^6$ psi and $\alpha_s = 6.5 \times 10^{-6}$/°F; for bronze, $E_b = 15 \times 10^6$ psi and $\alpha_b = 11 \times 10^{-6}$/°F.)



Solution 2.5-7 **Steel rod with bronze sleeve**



Elongation of the two outer parts of the bar

$\delta_1 = \alpha_s(\Delta T)(L_1 - L_2)$

$= (6.5 \times 10^{-6}/°\text{F})(500°\text{F})(36\text{ in.} - 12\text{ in.})$

$= 0.07800$ in.

Elongation of the middle part of the bar

The steel rod and bronze sleeve lengthen the same amount, so they are in the same condition as the bolt and sleeve of Example 2-8. Thus, we can calculate the elongation from Eq. (2-21):

$$\delta_2 = \frac{(\alpha_s E_s A_s + \alpha_b E_b A_b)(\Delta T)L_2}{E_s A_s + E_b A_b}$$

Substitute numerical values:

$\alpha_s = 6.5 \times 10^{-6}$/°F $\alpha_b = 11 \times 10^{-6}$/°F

$E_s = 30 \times 10^6$ psi $E_b = 15 \times 10^6$ psi

$d_1 = 1.0$ in.

$$A_s = \frac{\pi}{4}d_1^2 = 0.78540\text{ in.}^2$$

$d_2 = 1.25$ in.

$$A_b = \frac{\pi}{4}(d_2^2 - d_1^2) = 0.44179\text{ in.}^2$$

$\Delta T = 500°$F $L_2 = 12.0$ in.

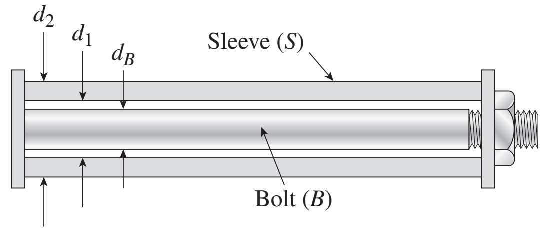
$\delta_2 = 0.04493$ in.

Total elongation

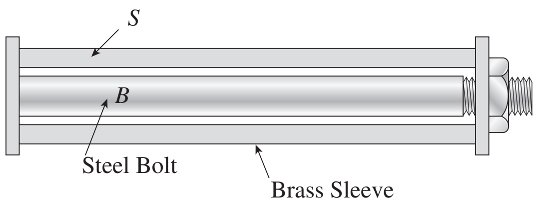
$\delta = \delta_1 + \delta_2 = 0.123$ in. ⟵

Problem 2.5-8 A brass sleeve S is fitted over a steel bolt B (see figure), and the nut is tightened until it is just snug. The bolt has a diameter $d_B = 25$ mm, and the sleeve has inside and outside diameters $d_1 = 26$ mm and $d_2 = 36$ mm, respectively.

Calculate the temperature rise ΔT that is required to produce a compressive stress of 25 MPa in the sleeve. (Use material properties as follows: for the sleeve, $\alpha_S = 21 \times 10^{-6}$/°C and $E_S = 100$ GPa; for the bolt, $\alpha_B = 10 \times 10^{-6}$/°C and $E_B = 200$ GPa.) (*Suggestion:* Use the results of Example 2-8.)



Solution 2.5-8 Brass sleeve fitted over a Steel bolt



Subscript S means "sleeve".

Subscript B means "bolt".

Use the results of Example 2-8.

σ_S = compressive force in sleeve

EQUATION (2-20A):

$$\sigma_S = \frac{(\alpha_S - \alpha_B)(\Delta T)E_S E_B A_B}{E_S A_S + E_B A_B} \text{ (Compression)}$$

SOLVE FOR ΔT:

$$\Delta T = \frac{\sigma_S(E_S A_S + E_B A_B)}{(\alpha_S - \alpha_B)E_S E_B A_B}$$

or

$$\Delta T = \frac{\sigma_S}{E_S(\alpha_S - \alpha_B)}\left(1 + \frac{E_S A_S}{E_B A_B}\right) \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_S = 25$ MPa

$d_2 = 36$ mm $\quad d_1 = 26$ mm $\quad d_B = 25$ mm

$E_S = 100$ GPa $\quad E_B = 200$ GPa

$\alpha_S = 21 \times 10^{-6}$/°C $\quad \alpha_B = 10 \times 10^{-6}$/°C

$$A_S = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(620 \text{ mm}^2)$$

$$A_B = \frac{\pi}{4}(d_B)^2 = \frac{\pi}{4}(625 \text{ mm}^2)$$

$$1 + \frac{E_S A_S}{E_B A_B} = 1.496$$

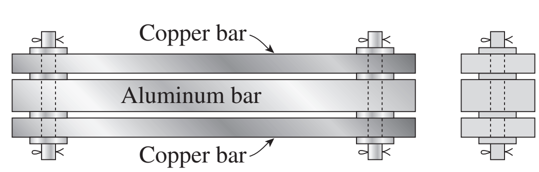
$$\Delta T = \frac{25 \text{ MPa } (1.496)}{(100 \text{ GPa})(11 \times 10^{-6}/°\text{C})}$$

$\Delta T = 34°\text{C} \longleftarrow$

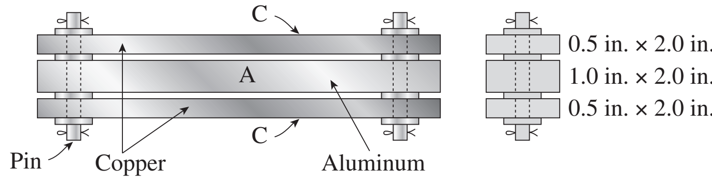
(Increase in temperature)

Problem 2.5-9 Rectangular bars of copper and aluminum are held by pins at their ends, as shown in the figure. Thin spacers provide a separation between the bars. The copper bars have cross-sectional dimensions 0.5 in. × 2.0 in., and the aluminum bar has dimensions 1.0 in. × 2.0 in.

Determine the shear stress in the 7/16 in. diameter pins if the temperature is raised by 100°F. (For copper, $E_c = 18{,}000$ ksi and $\alpha_c = 9.5 \times 10^{-6}$/°F; for aluminum, $E_a = 10{,}000$ ksi and $\alpha_a = 13 \times 10^{-6}$/°F.) *Suggestion:* Use the results of Example 2-8.



Solution 2.5-9 **Rectangular bars held by pins**



Diameter of pin: $d_P = \frac{7}{16}$ in. = 0.4375 in.

Area of pin: $A_P = \frac{\pi}{4} d_P^2 = 0.15033 \text{ in.}^2$

Area of two copper bars: $A_c = 2.0 \text{ in.}^2$

Area of aluminum bar: $A_a = 2.0 \text{ in.}^2$

$\Delta T = 100°\text{F}$

Copper: $E_c = 18{,}000$ ksi $\qquad \alpha_c = 9.5 \times 10^{-6}/°\text{F}$

Aluminum: $E_a = 10{,}000$ ksi $\qquad \alpha_a = 13 \times 10^{-6}/°\text{F}$

Use the results of Example 2-8.

Find the forces P_a and P_c in the aluminum bar and copper bar, respectively, from Eq. (2-19).

Replace the subscript "S" in that equation by "a" (for aluminum) and replace the subscript "B" by "c" (for copper), because α for aluminum is larger than α for copper.

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T) E_a A_a E_c A_c}{E_a A_a + E_c A_c}$$

Note that P_a is the compressive force in the aluminum bar and P_c is the combined tensile force in the two copper bars.

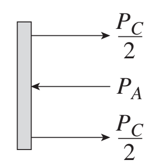
$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T) E_c A_c}{1 + \dfrac{E_c A_c}{E_a A_a}}$$

SUBSTITUTE NUMERICAL VALUES:

$$P_a = P_c = \frac{(3.5 \times 10^{-6}/°\text{F})(100°\text{F})(18{,}000 \text{ ksi})(2 \text{ in.}^2)}{1 + \left(\dfrac{18}{10}\right)\left(\dfrac{2.0}{2.0}\right)}$$

$$= 4{,}500 \text{ lb}$$

FREE-BODY DIAGRAM OF PIN AT THE LEFT END



V = shear force in pin

$= P_c/2$

$= 2{,}250$ lb

τ = average shear stress on cross section of pin

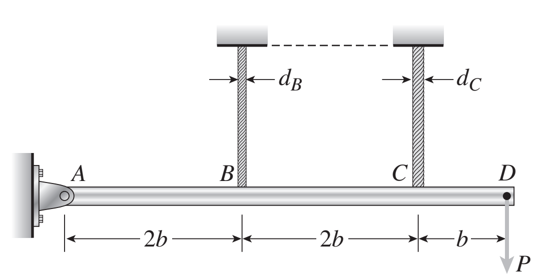
$$\tau = \frac{V}{A_P} = \frac{2{,}250 \text{ lb}}{0.15033 \text{ in.}^2}$$

$\tau = 15.0$ ksi $\longleftarrow$

Problem 2.5-10 A rigid bar $ABCD$ is pinned at end A and supported by two cables at points B and C (see figure). The cable at B has nominal diameter $d_B = 12$ mm and the cable at C has nominal diameter $d_C = 20$ mm. A load P acts at end D of the bar.

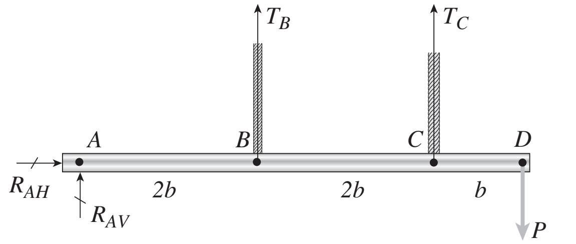
What is the allowable load P if the temperature rises by 60°C and each cable is required to have a factor of safety of at least 5 against its ultimate load?

(*Note:* The cables have effective modulus of elasticity $E = 140$ GPa and coefficient of thermal expansion $\alpha = 12 \times 10^{-6}/°\text{C}$. Other properties of the cables can be found in Table 2-1, Section 2.2.)



Solution 2.5-10 Rigid bar supported by two cables

FREE-BODY DIAGRAM OF BAR ABCD



T_B = force in cable B $\quad$ T_C = force in cable C

$d_B = 12$ mm $\quad$ $d_C = 20$ mm

From Table 2-1:

$A_B = 76.7\text{ mm}^2$ $\quad$ $E = 140$ GPa

$\Delta T = 60°C$ $\quad$ $A_C = 173\text{ mm}^2$

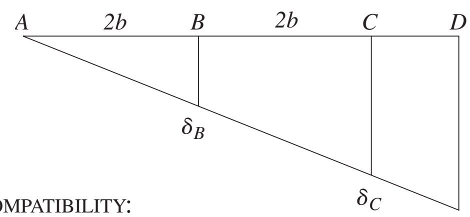
$\alpha = 12 \times 10^{-6}/°C$

EQUATION OF EQUILIBRIUM

$\Sigma M_A = 0 \quad \circlearrowleft + \quad T_B(2b) + T_C(4b) - P(5b) = 0$

or $\quad 2T_B + 4T_C = 5P \qquad$ (Eq. 1)

DISPLACEMENT DIAGRAM



COMPATIBILITY:

$\delta_C = 2\delta_B \qquad$ (Eq. 2)

FORCE-DISPLACEMENT AND TEMPERATURE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{T_B L}{EA_B} + \alpha(\Delta T)L \qquad \text{(Eq. 3)}$$

$$\delta_C = \frac{T_C L}{EA_C} + \alpha(\Delta T)L \qquad \text{(Eq. 4)}$$

SUBSTITUTE EQS. (3) AND (4) INTO EQ. (2):

$$\frac{T_C L}{EA_C} + \alpha(\Delta T)L = \frac{2T_B L}{EA_B} + 2\alpha(\Delta T)L$$

or

$$2T_B A_C - T_C A_B = -E\alpha(\Delta T)A_B A_C \qquad \text{(Eq. 5)}$$

SUBSTITUTE NUMERICAL VALUES INTO EQ. (5):

$T_B(346) - T_C(76.7) = -1{,}338{,}000 \qquad$ (Eq. 6)

in which T_B and T_C have units of newtons.

SOLVE SIMULTANEOUSLY EQS. (1) AND (6):

$T_B = 0.2494\ P - 3{,}480 \qquad$ (Eq. 7)

$T_C = 1.1253\ P + 1{,}740 \qquad$ (Eq. 8)

in which P has units of newtons.

SOLVE EQS. (7) AND (8) FOR THE LOAD P:

$P_B = 4.0096\ T_B + 13{,}953 \qquad$ (Eq. 9)

$P_C = 0.8887\ T_C - 1{,}546 \qquad$ (Eq. 10)

ALLOWABLE LOADS

From Table 2-1:

$(T_B)_{ULT} = 102{,}000$ N $\quad$ $(T_C)_{ULT} = 231{,}000$ N

Factor of safety = 5

$(T_B)_{\text{allow}} = 20{,}400$ N $\quad$ $(T_C)_{\text{allow}} = 46{,}200$ N

From Eq. (9): $P_B = (4.0096)(20{,}400\text{ N}) + 13{,}953\text{ N}$
$= 95{,}700$ N

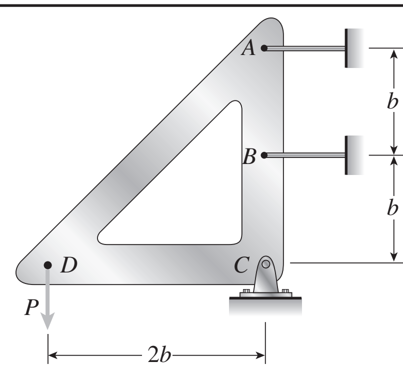
From Eq. (10): $P_C = (0.8887)(46{,}200\text{ N}) - 1546\text{ N}$
$= 39{,}500$ N

Cable C governs.

$P_{\text{allow}} = 39.5$ kN ⟵

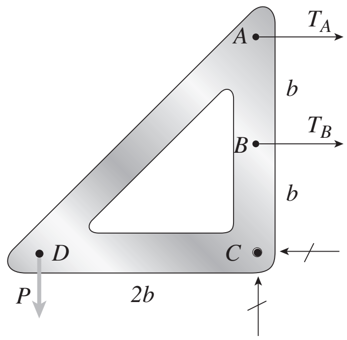
Problem 2.5-11 A rigid triangular frame is pivoted at C and held by two identical horizontal wires at points A and B (see figure). Each wire has axial rigidity $EA = 120$ k and coefficient of thermal expansion $\alpha = 12.5 \times 10^{-6}/°F$.

(a) If a vertical load $P = 500$ lb acts at point D, what are the tensile forces T_A and T_B in the wires at A and B, respectively?
(b) If, while the load P is acting, both wires have their temperatures raised by 180°F, what are the forces T_A and T_B?
(c) What further increase in temperature will cause the wire at B to become slack?



Solution 2.5-11 Triangular frame held by two wires

FREE-BODY DIAGRAM OF FRAME

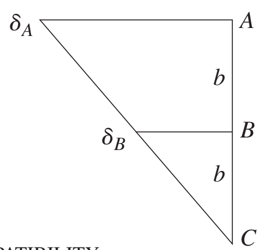


EQUATION OF EQUILIBRIUM

$\Sigma M_C = 0$ ↶+ ↷−

$P(2b) - T_A(2b) - T_B(b) = 0$ or $2T_A + T_B = 2P$ (Eq. 1)

DISPLACEMENT DIAGRAM



EQUATION OF COMPATIBILITY

$\delta_A = 2\delta_B$ (Eq. 2)

(a) LOAD P ONLY

Force-displacement relations:

$$\delta_A = \frac{T_A L}{EA} \qquad \delta_B = \frac{T_B L}{EA} \qquad \text{(Eq. 3, 4)}$$

(L = length of wires at A and B.)

Substitute (3) and (4) into Eq. (2):

$$\frac{T_A L}{EA} = \frac{2T_B L}{EA}$$

or $T_A = 2T_B$ (Eq. 5)

Solve simultaneously Eqs. (1) and (5):

$$T_A = \frac{4P}{5} \qquad T_B = \frac{2P}{5} \qquad \text{(Eqs. 6, 7)}$$

Numerical values:

$P = 500$ lb

$\therefore\ T_A = 400$ lb $\quad T_B = 200$ lb ⟵

(b) LOAD P AND TEMPERATURE INCREASE ΔT

Force-displacement and temperature-displacement relations:

$$\delta_A = \frac{T_A L}{EA} + \alpha(\Delta T)L \qquad \text{(Eq. 8)}$$

$$\delta_B = \frac{T_B L}{EA} + \alpha(\Delta T)L \qquad \text{(Eq. 9)}$$

Substitute (8) and (9) into Eq. (2):

$$\frac{T_A L}{EA} + \alpha(\Delta T)L = \frac{2T_B L}{EA} + 2\alpha(\Delta T)L$$

or $T_A - 2T_B = EA\alpha(\Delta T)$ (Eq. 10)

Solve simultaneously Eqs. (1) and (10):

$$T_A = \frac{1}{5}[4P + EA\alpha(\Delta T)] \qquad \text{(Eq. 11)}$$

$$T_B = \frac{2}{5}[P - EA\alpha(\Delta T)] \qquad \text{(Eq. 12)}$$

Substitute numerical values:

$P = 500$ lb $\qquad EA = 120{,}000$ lb

$\Delta T = 180°$F

$\alpha = 12.5 \times 10^{-6}/°$F

$$T_A = \frac{1}{5}(2000 \text{ lb} + 270 \text{ lb}) = 454 \text{ lb} \longleftarrow$$

$$T_B = \frac{2}{5}(500 \text{ lb} - 270 \text{ lb}) = 92 \text{ lb} \longleftarrow$$

(c) WIRE B BECOMES SLACK

Set $T_B = 0$ in Eq. (12):

$P = EA\alpha(\Delta T)$

or

$$\Delta T = \frac{P}{EA\alpha} = \frac{500 \text{ lb}}{(120{,}000 \text{ lb})(12.5 \times 10^{-6}/°\text{F})}$$

$= 333.3°$F

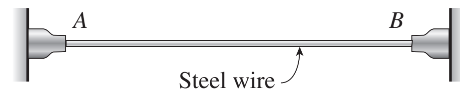
Further increase in temperature:

$\Delta T = 333.3°\text{F} - 180°\text{F}$

$= 153°$F ⟵

Misfits and Prestrains

Problem 2.5-12 A steel wire *AB* is stretched between rigid supports (see figure). The initial prestress in the wire is 42 MPa when the temperature is 20°C.



(a) What is the stress σ in the wire when the temperature drops to 0°C?
(b) At what temperature T will the stress in the wire become zero? (Assume $\alpha = 14 \times 10^{-6}$/°C and $E = 200$ GPa.)

Solution 2.5-12 **Steel wire with initial prestress**

Initial prestress: $\sigma_1 = 42$ MPa

Initial temperature: $T_1 = 20°\text{C}$

$E = 200$ GPa

$\alpha = 14 \times 10^{-6}/°\text{C}$

(a) STRESS Σ WHEN TEMPERATURE DROPS TO 0°C

$T_2 = 0°\text{C}$ $\qquad \Delta T = 20°\text{C}$

Note: Positive ΔT means a *decrease* in temperature and an *increase* in the stress in the wire.

Negative ΔT means an *increase* in temperature and a *decrease* in the stress.

Stress σ equals the initial stress σ_1 plus the additional stress σ_2 due to the temperature drop.

From Eq. (2-18): $\sigma_2 = E\alpha(\Delta T)$

$$\sigma = \sigma_1 + \sigma_2 = \sigma_1 + E\alpha(\Delta T)$$
$$= 42\text{ MPa} + (200\text{ GPa})(14 \times 10^{-6}/°\text{C})(20°\text{C})$$
$$= 42\text{ MPa} + 56\text{ MPa} = 98\text{ MPa} \longleftarrow$$

(b) TEMPERATURE WHEN STRESS EQUALS ZERO

$$\sigma = \sigma_1 + \sigma_2 = 0 \qquad \sigma_1 + E\alpha(\Delta T) = 0$$

$$\Delta T = -\frac{\sigma_1}{E\alpha}$$

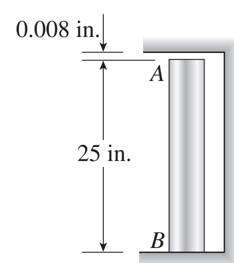
(Negative means increase in temp.)

$$\Delta T = -\frac{42\text{ MPa}}{(200\text{ GPa})(14 \times 10^{-6}/°\text{C})} = -15°\text{C}$$

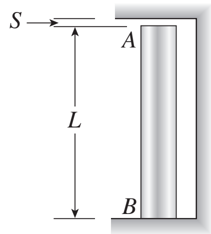
$$T = 20°\text{C} + 15°\text{C} = 35°\text{C} \longleftarrow$$

Problem 2.5-13 A copper bar *AB* of length 25 in. is placed in position at room temperature with a gap of 0.008 in. between end *A* and a rigid restraint (see figure).

Calculate the axial compressive stress σ_c in the bar if the temperature rises 50°F. (For copper, use $\alpha = 9.6 \times 10^{-6}$/°F and $E = 16 \times 10^6$ psi.)



Solution 2.5-13 Bar with a gap



$L = 25$ in.

$S = 0.008$ in.

$\Delta T = 50°F$ (increase)

$\alpha = 9.6 \times 10^{-6}/°F$

$E = 16 \times 10^6$ psi

δ = elongation of the bar if it is free to expand

$= \alpha(\Delta T)L$

δ_C = elongation that is prevented by the support

$= \alpha(\Delta T)L - S$

ε_C = strain in the bar due to the restraint

$= \delta_C/L$

σ_c = stress in the bar

$$= E\varepsilon_C = \frac{E\delta_C}{L} = \frac{E}{L}[\alpha(\Delta T)L - S] \longleftarrow$$

Note: This result is valid only if $\alpha(\Delta T)L \geq S$. (Otherwise, the gap is not closed).

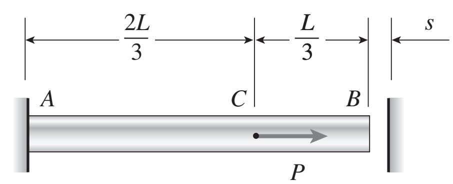
Substitute numerical values:

$$\sigma_c = \frac{16 \times 10^6 \text{ psi}}{25 \text{ in.}}[(9.6 \times 10^{-6}/°F)(50°F)(25 \text{ in.})$$

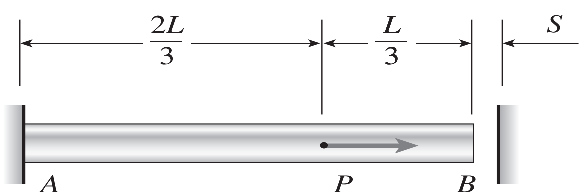
$$- 0.008 \text{ in.}] = 2{,}560 \text{ psi} \longleftarrow$$

Problem 2.5-14 A bar AB having length L and axial rigidity EA is fixed at end A (see figure). At the other end a small gap of dimension s exists between the end of the bar and a rigid surface. A load P acts on the bar at point C, which is two-thirds of the length from the fixed end.

If the support reactions produced by the load P are to be equal in magnitude, what should be the size s of the gap?



Solution 2.5-14 Bar with a gap (load P)



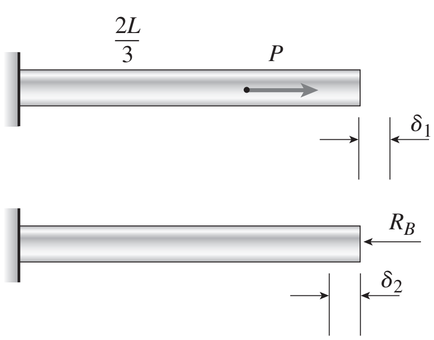
L = length of bar

S = size of gap

EA = axial rigidity

Reactions must be equal; find S.

FORCE-DISPLACEMENT RELATIONS



$$\delta_1 = \frac{P(\frac{2L}{3})}{EA}$$

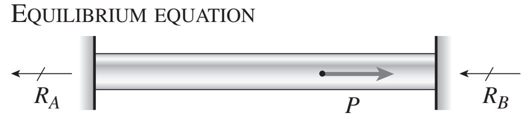
$$\delta_2 = \frac{R_B L}{EA}$$

COMPATIBILITY EQUATION

$\delta_1 - \delta_2 = S$ or

$$\frac{2PL}{3EA} - \frac{R_B L}{EA} = S \qquad \text{(Eq. 1)}$$

EQUILIBRIUM EQUATION



R_A = reaction at end A (to the left)

R_B = reaction at end B (to the left)

$P = R_A + R_B$

Reactions must be equal.

$\therefore R_A = R_B \qquad P = 2R_B \qquad R_B = \frac{P}{2}$

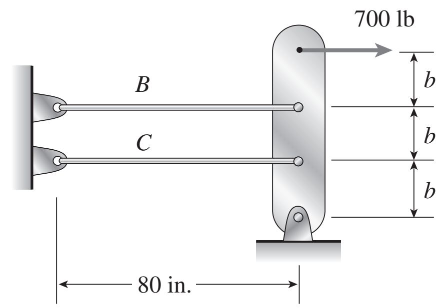
Substitute for R_B in Eq. (1):

$$\frac{2PL}{3EA} - \frac{PL}{2EA} = S \quad \text{or} \quad S = \frac{PL}{6EA} \longleftarrow$$

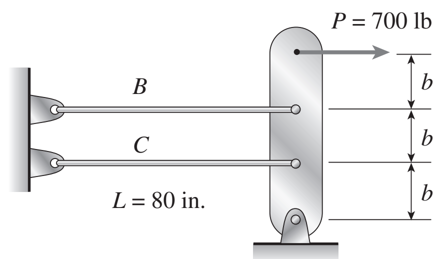
NOTE: The gap closes when the load reaches the value $P/4$. When the load reaches the value P, equal to $6EAs/L$, the reactions are equal ($R_A = R_B = P/2$). When the load is between $P/4$ and P, R_A is greater than R_B. If the load exceeds P, R_B is greater than R_A.

Problem 2.5-15 Wires B and C are attached to a support at the left-hand end and to a pin-supported rigid bar at the right-hand end (see figure). Each wire has cross-sectional area $A = 0.03$ in.^{2} and modulus of elasticity $E = 30 \times 10^6$ psi. When the bar is in a vertical position, the length of each wire is $L = 80$ in. However, before being attached to the bar, the length of wire B was 79.98 in. and of wire C was 79.95 in.

Find the tensile forces T_B and T_C in the wires under the action of a force $P = 700$ lb acting at the upper end of the bar.



Solution 2.5-15 **Wires B and C attached to a bar**



$P = 700$ lb

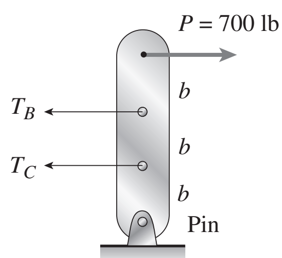
$A = 0.03$ in.^{2}

$E = 30 \times 10^6$ psi

$L_B = 79.98$ in.

$L_C = 79.95$ in.

EQUILIBRIUM EQUATION



$\Sigma M_{\text{pin}} = 0$

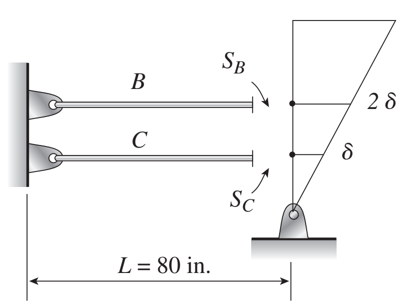
$T_C(b) + T_B(2b) = P(3b)$

$2T_B + T_C = 3P$ (Eq. 1)

DISPLACEMENT DIAGRAM

$S_B = 80 \text{ in.} - L_B = 0.02 \text{ in.}$

$S_C = 80 \text{ in.} - L_C = 0.05 \text{ in.}$



Elongation of wires:

$\delta_B = S_B + 2\delta$ (Eq. 2)

$\delta_C = S_C + \delta$ (Eq. 3)

FORCE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{T_B L}{EA} \qquad \delta_C = \frac{T_C L}{EA} \qquad \text{(Eqs. 4, 5)}$$

SOLUTION OF EQUATIONS

Combine Eqs. (2) and (4):

$$\frac{T_B L}{EA} = S_B + 2\delta \qquad \text{(Eq. 6)}$$

Combine Eqs. (3) and (5):

$$\frac{T_C L}{EA} = S_C + \delta \qquad \text{(Eq. 7)}$$

Eliminate δ between Eqs. (6) and (7):

$$T_B - 2T_C = \frac{EAS_B}{L} - \frac{2EAS_C}{L} \qquad \text{(Eq. 8)}$$

Solve simultaneously Eqs. (1) and (8):

$$T_B = \frac{6P}{5} + \frac{EAS_B}{5L} - \frac{2EAS_C}{5L} \longleftarrow$$

$$T_C = \frac{3P}{5} - \frac{2EAS_B}{5L} + \frac{4EAS_C}{5L} \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\frac{EA}{5L} = 2250 \text{ lb/in.}$$

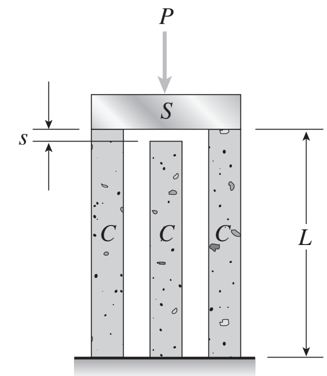
$T_B = 840 \text{ lb} + 45 \text{ lb} - 225 \text{ lb} = 660 \text{ lb} \longleftarrow$

$T_C = 420 \text{ lb} - 90 \text{ lb} + 450 \text{ lb} = 780 \text{ lb} \longleftarrow$

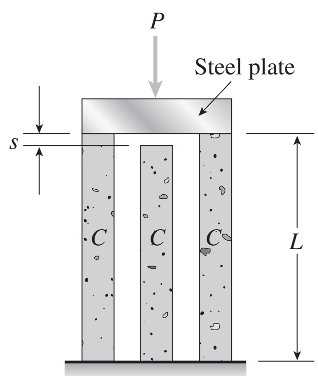
(Both forces are positive, which means tension, as required for wires.)

Problem 2.5-16 A rigid steel plate is supported by three posts of high-strength concrete each having an effective cross-sectional area $A = 40{,}000\ \text{mm}^2$ and length $L = 2$ m (see figure). Before the load P is applied, the middle post is shorter than the others by an amount $s = 1.0$ mm.

Determine the maximum allowable load P_{allow} if the allowable compressive stress in the concrete is σ_{allow} 5 20 MPa. (Use $E = 30$ GPa for concrete.)



Solution 2.5-16 **Plate supported by three posts**



S = size of gap = 1.0 mm

L = length of posts = 2.0 m

$A = 40{,}000\ \text{mm}^2$

$\sigma_{allow} = 20$ MPa

$E = 30$ GPa

C = concrete post

DOES THE GAP CLOSE?

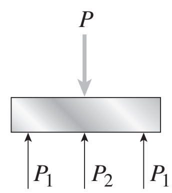
Stress in the two outer posts when the gap is just closed:

$$\sigma = E\varepsilon = E\left(\frac{S}{L}\right) = (30\ \text{GPa})\left(\frac{1.0\ \text{mm}}{2.0\ \text{m}}\right)$$

$$= 15\ \text{MPa}$$

Since this stress is less than the allowable stress, the allowable force P will close the gap.

EQUILIBRIUM EQUATION



$$2P_1 + P_2 = P \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION

δ_1 = shortening of outer posts

δ_2 = shortening of inner post

$$\delta_1 = \delta_2 + S \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{P_1 L}{EA} \qquad \delta_2 = \frac{P_2 L}{EA} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_1 L}{EA} = \frac{P_2 L}{EA} + S \quad \text{or} \quad P_1 - P_2 = \frac{EAS}{L} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P = 3P_1 - \frac{EAS}{L}$$

By inspection, we know that P_1 is larger than P_2. Therefore, P_1 will control and will be equal to $\sigma_{allow} A$.

$$P_{allow} = 3\sigma_{allow} A - \frac{EAS}{L}$$

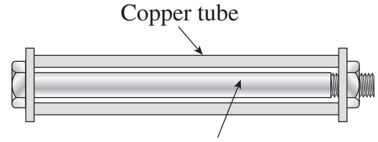
$$= 2400\ \text{kN} - 600\ \text{kN} = 1800\ \text{kN}$$

$$= 1.8\ \text{MN} \longleftarrow$$

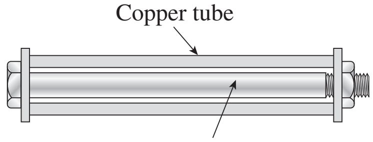
Problem 2.5-17 A copper tube is fitted around a steel bolt and the nut is turned until it is just snug (see figure). What stresses σ_s and σ_c will be produced in the steel and copper, respectively, if the bolt is now tightened by a quarter turn of the nut?

The copper tube has length $L = 16$ in. and cross-sectional area $A_c = 0.6$ in.^{2}, and the steel bolt has cross-sectional area $A_s = 0.2$ in.^{2} The pitch of the threads of the bolt is $p = 52$ mils (a mil is one-thousandth of an inch). Also, the moduli of elasticity of the steel and copper are $E_s = 30 \times 10^6$ psi and $E_c = 16 \times 10^6$ psi, respectively.

Note: The pitch of the threads is the distance advanced by the nut in one complete turn (see Eq. 2-22).



Solution 2.5-17 Steel bolt and copper tube



$L = 16$ in.

$P = 52$ mils $= 0.052$ in.

$n = \frac{1}{4}$ (See Eq. 2-22)

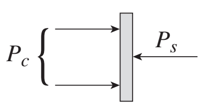
Steel bolt: $A_s = 0.2$ in.^{2}

$E_s = 30 \times 10^6$ psi

Copper tube: $A_c = 0.6$ in.^{2}

$E_c = 16 \times 10^6$ psi

EQUILIBRIUM EQUATION

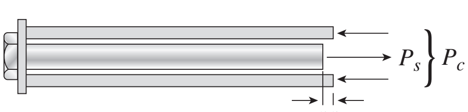


P_s = tensile force in steel bolt

P_c = compressive force in copper tube

$P_c = P_s$ (Eq. 1)

COMPATIBILITY EQUATION



δ_c = shortening of copper tube

δ_s = elongation of steel bolt

$\delta_c + \delta_s = np$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$\delta_c = \frac{P_c L}{E_c A_c} \quad \delta_s = \frac{P_s L}{E_s A_s} \qquad \text{(Eq. 3, Eq. 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_c L}{E_c A_c} + \frac{P_s L}{E_s A_s} = np \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P_s = P_c = \frac{npE_s A_s E_c A_c}{L(E_s A_s + E_c A_c)} \qquad \text{(Eq. 6)}$$

Substitute numerical values:

$P_s = P_c = 3{,}000$ lb

STRESSES

Steel bolt:

$$\sigma_s = \frac{P_s}{A_s} = \frac{3{,}000 \text{ lb}}{0.2 \text{ in.}^2} = 15 \text{ ksi (Tension)} \longleftarrow$$

Copper tube:

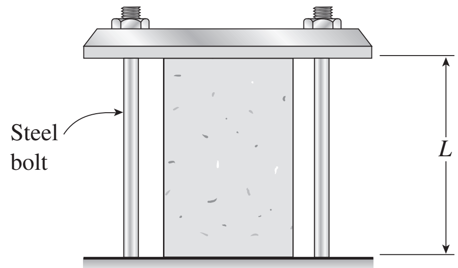
$$\sigma_c = \frac{P_c}{A_c} = \frac{3{,}000 \text{ lb}}{0.6 \text{ in.}^2}$$

$$= 5 \text{ ksi (compression)} \longleftarrow$$

Problem 2.5-18 A plastic cylinder is held snugly between a rigid plate and a foundation by two steel bolts (see figure).

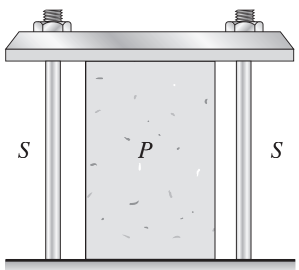
Determine the compressive stress σ_p in the plastic when the nuts on the steel bolts are tightened by one complete turn.

Data for the assembly are as follows: length $L = 200$ mm, pitch of the bolt threads $p = 1.0$ mm, modulus of elasticity for steel $E_s = 200$ GPa, modulus of elasticity for the plastic $E_p = 7.5$ GPa, cross-sectional area of one bolt $A_s = 36.0$ mm^2, and cross-sectional area of the plastic cylinder $A_p = 960$ mm^2.



Probs. 2.5-18 and 2.5-19

Solution 2.5-18 **Plastic cylinder and two steel bolts**



$L = 200$ mm

$P = 1.0$ mm

$E_s = 200$ GPa

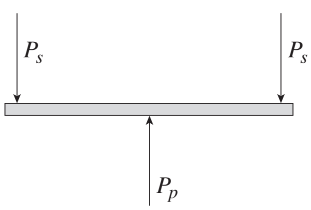
$A_s = 36.0$ mm^2 (for one bolt)

$E_p = 7.5$ GPa

$A_p = 960$ mm^2

$n = 1$ (See Eq. 2-22)

EQUILIBRIUM EQUATION

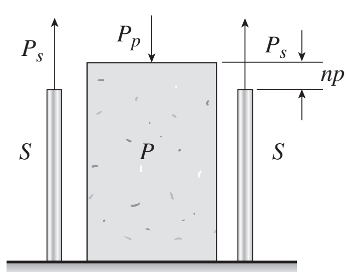


P_s = tensile force in one steel bolt

P_p = compressive force in plastic cylinder

$$P_p = 2P_s \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION



δ_s = elongation of steel bolt

δ_p = shortening of plastic cylinder

$$\delta_s + \delta_p = np \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_p = \frac{P_p L}{E_p A_p} \qquad \text{(Eq. 3, Eq. 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2npE_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2\,np\,E_s A_s E_p}{L(E_p A_p + 2E_s A_s)} \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$N = E_s A_s E_p = 54.0 \times 10^{15}\ \text{N}^2/\text{mm}^2$$

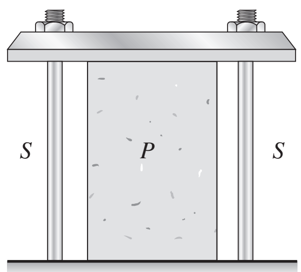
$$D = E_p A_p + 2E_s A_s = 21.6 \times 10^6\ \text{N}$$

$$\sigma_p = \frac{2np}{L}\left(\frac{N}{D}\right) = \frac{2(1)(1.0\ \text{mm})}{200\ \text{mm}}\left(\frac{N}{D}\right)$$

$$= 25.0\ \text{MPa} \longleftarrow$$

Problem 2.5-19 Solve the preceding problem if the data for the assembly are as follows: length $L = 10$ in., pitch of the bolt threads $p = 0.058$ in., modulus of elasticity for steel $E_s = 30 \times 10^6$ psi, modulus of elasticity for the plastic $E_p = 500$ ksi, cross-sectional area of one bolt $A_s = 0.06$ in.^{2}, and cross-sectional area of the plastic cylinder $A_p = 1.5$ in.^{2}

Solution 2.5-19 Plastic cylinder and two steel bolts



$L = 10$ in.

$P = 0.058$ in.

$E_s = 30 \times 10^6$ psi

$A_s = 0.06$ in.^{2} (for one bolt)

$E_p = 500$ ksi

$A_p = 1.5$ in.^{2}

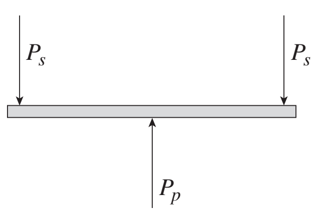
$n = 1$ (see Eq. 2-22)

EQUILIBRIUM EQUATION

P_s = tensile force in one steel bolt

P_p = compressive force in plastic cylinder

$P_p = 2P_s$ (Eq. 1)

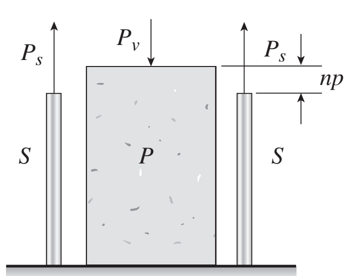


COMPATIBILITY EQUATION

δ_s = elongation of steel bolt

δ_p = shortening of plastic cylinder

$\delta_s + \delta_p = np$ (Eq. 2)



FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \qquad \delta_p = \frac{P_p L}{E_p A_p} \qquad \text{(Eq. 3, Eq. 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2\,np\,E_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2\,np\,E_s A_s E_p}{L(E_p A_p + 2E_s A_s)} \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$N = E_s A_s E_p = 900 \times 10^9 \text{ lb}^2/\text{in.}^2$$

$$D = E_p A_p + 2E_s A_s = 4350 \times 10^3 \text{ lb}$$

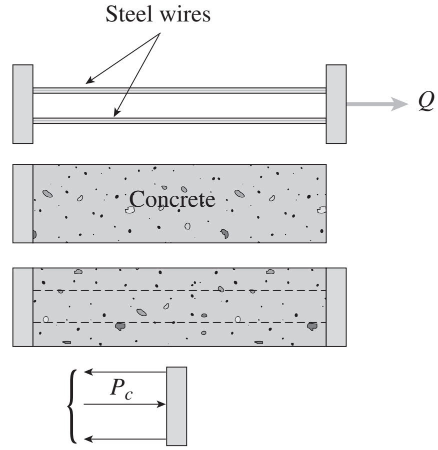
$$\sigma_P = \frac{2np}{L}\left(\frac{N}{D}\right) = \frac{2(1)(0.058 \text{ in.})}{10 \text{ in.}}\left(\frac{N}{D}\right)$$

$$= 2400 \text{ psi} \longleftarrow$$

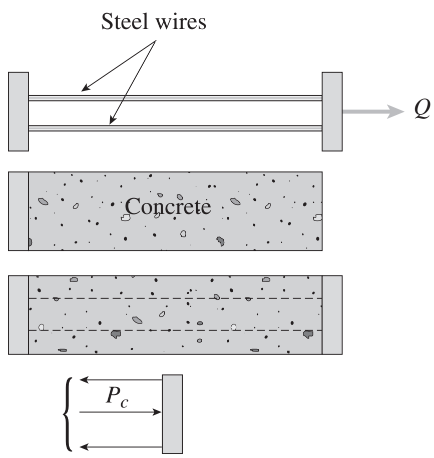
Problem 2.5-20 Prestressed concrete beams are sometimes manufactured in the following manner. High-strength steel wires are stretched by a jacking mechanism that applies a force Q, as represented schematically in part (a) of the figure. Concrete is then poured around the wires to form a beam, as shown in part (b).

After the concrete sets properly, the jacks are released and the force Q is removed [see part (c) of the figure]. Thus, the beam is left in a prestressed condition, with the wires in tension and the concrete in compression.

Let us assume that the prestressing force Q produces in the steel wires an initial stress $\sigma_0 = 620$ MPa. If the moduli of elasticity of the steel and concrete are in the ratio 12:1 and the cross-sectional areas are in the ratio 1:50, what are the final stresses σ_s and σ_c in the two materials?



Solution 2.5-20 **Prestressed concrete beam**



L = length

σ_0 = initial stress in wires

$= \dfrac{Q}{A_s} = 620$ MPa

A_s = total area of steel wires

A_c = area of concrete

$= 50\,A_s$

$E_s = 12\,E_c$

P_s = final tensile force in steel wires

P_c = final compressive force in concrete

EQUILIBRIUM EQUATION

$$P_s = P_c \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION AND FORCE-DISPLACEMENT RELATIONS

δ_1 = initial elongation of steel wires

$$= \frac{QL}{E_s A_s} = \frac{\sigma_0 L}{E_s}$$

δ_2 = final elongation of steel wires

$$= \frac{P_s L}{E_s A_s}$$

δ_3 = shortening of concrete

$$= \frac{P_c L}{E_c A_c}$$

$$\delta_1 - \delta_2 = \delta_3 \quad \text{or} \quad \frac{\sigma_0 L}{E_s} - \frac{P_s L}{E_s A_s} = \frac{P_c L}{E_c A_c} \quad \text{(Eq. 2, Eq. 3)}$$

Solve simultaneously Eqs. (1) and (3):

$$P_s = P_c = \frac{\sigma_0 A_s}{1 + \dfrac{E_s A_s}{E_c A_c}}$$

STRESSES

$$\sigma_s = \frac{P_s}{A_s} = \frac{\sigma_0}{1 + \dfrac{E_s A_s}{E_c A_c}} \longleftarrow$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{\sigma_0}{\dfrac{A_c}{A_s} + \dfrac{E_s}{E_c}} \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_0 = 620 \text{ MPa} \qquad \frac{E_s}{E_c} = 12 \qquad \frac{A_s}{A_c} = \frac{1}{50}$$

$$\sigma_s = \frac{620 \text{ MPa}}{1 + \dfrac{12}{50}} = 500 \text{ MPa (Tension)} \longleftarrow$$

$$\sigma_c = \frac{620 \text{ MPa}}{50 + 12} = 10 \text{ MPa (Compression)} \longleftarrow$$